Characterization and Prediction of the Compressive Stress-Strain Relationship of Layered Arrays of Spongy Baked Goods

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ABSTRACT

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The sigmoid compressive stress-strain curves of crumbs from three types of bread, pound cake, and double-layered arrays of these materials (0–70% deformation) are described by two kinds of three-parameter mathematical models. The model structures were especially selected so that they could be transformed algebraically from stress-strain to strain-stress relationships or be determined directly from the strain-stress data by nonlinear regression. The model parameters enabled calculation of a specimen's deformation-force relationship with any given dimensions. When an in-series array of objects having the same cross-sectional area is deformed uniaxially, the force along the array is the same and the deformation the sum of that of its components. Since sponge compression is not accompanied by a significant lateral expansion, the effects of friction between the layers can be neglected. This enabled the prediction of the force-deformation and stress-strain relationships of double-layered arrays with reasonable accuracy from the compression parameters of the components' material.

A typical mechanical feature of most sponges and solid foams is the sigmoid shape of their compressive stress-strain relationship (Gibson and Ashby 1988). Its three regions are a manifestation of three deformation mechanisms, namely and in order of dominancy, elastic or quasi-elastic deformation of the intact cellular structure, collapse by buckling and/or fracture of the cell walls, and densification, i.e., compaction of the collapsed cell wall material (Ashby 1983). Cellular solid mechanics have recently been applied to a spongy baked good by Attenburrow et al (1989). An empirical model for mathematical characterization of the compressive stress-strain behavior up to around 75% deformation was described by Peleg et al (1989). It also was shown that this model is not unique and that several three- or four-parameter models, of very different mathematical constructions, are just as applicable to bread as to polymeric foams (Swyngedau et al 1991b).

Another characteristic of most cellular solids is that their compression is not accompanied by lateral expansion. If a layered array of sponges is compressed (Fig. 1), frictional effects in the contact region are expected to be of a negligible magnitude (Swyngedau et al 1991a). When all of the layers have the same cross-sectional area, the force along the array is the same and the deformation is the sum of the deformations of the individual layers. Consequently, if the stress-strain (σ vs. ϵ) relationship of the material of each component, i, is known, it can be used to calculate the force-deformation relationship (F vs. ΔH_i) of a layer with any given dimensions (original height H_{0i} and area A) by the simple transformations

$$F = A\sigma \tag{1}$$

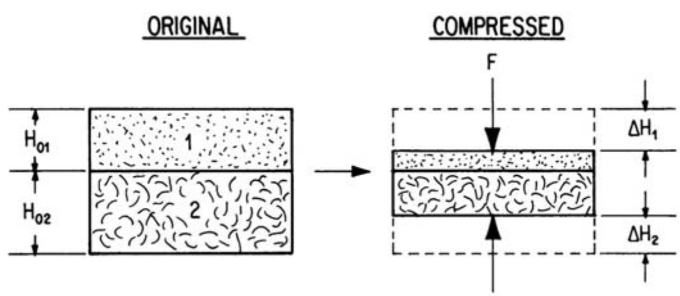

Fig. 1. Geometry of a uniaxially compressed double-layered array.

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and

$$\Delta H_i = H_{0i}\epsilon_{Ei}, \tag{2}$$

when the strain is expressed as the engineering strain, i.e., $\epsilon_E = \Delta H/H_0$, or

$$\Delta H_i = H_{0i}[1 - \exp - (\epsilon_{Hi})], \tag{3}$$

when the strain is expressed as Hencky's strain, i.e., $\epsilon_H = \ln[H_0/(H_0 - \Delta H)]$. Because at large deformations the relative effect of the same amount of displacement is not constant, the use of Hencky's (also known as natural or "true") strain is preferable (Marin 1965). The shape of the curve is altered to some extent when used to describe the stress-strain relationships of sponges, but the fit of the various models is unaffected (see below and Swyngedau et al 1991a,b).

The force-deformation of an array of given area (A_a), when the height (thickness) of each layer also is given, now can be calculated from

$$\sigma = F/A_a \tag{4}$$

and

$$\epsilon_{Ea} = \frac{\sum_{i=1}^{n} \Delta H_i}{\sum_{i=1}^{n} H_{0i}}, \tag{5}$$

where ϵ_{Ea} is the engineering strain of the array, or

$$\epsilon_{Ha} = \ln\left[\frac{\sum_{i=1}^{n} H_{0i}}{\sum_{i=1}^{n} H_{0i} - \sum_{i=1}^{n} \Delta H_i}\right] \tag{6}$$

where ϵ_{Ha} is the Hencky's strain of the array. In both equations the subscript i represents an individual layer $i = 1,2,\ldots$ and n the total number of layers in the array. If the strain-stress relationship of all of the components is given as certain types of explicit algebraic expressions (see below), then the construction of the strain-stress relationship using equations 4 and 5 or 6 can be done algebraically. Otherwise, it will require a more lengthy and cumbersome numerical solution (Roy et al 1988).

The practicality of a procedure based on such expressions was recently demonstrated in various combinations of polymeric foams (Swyngedau et al 1991a), and we tested its applicability to arrays of spongy baked goods. The objectives of this work were to demonstrate the method with experimental compression data of selected breads and a spongy cake and to determine whether the underlying assumptions on which the method is based are valid not only in synthetic foams but also in bakery products.

TABLE I
Density and Thickness of Layers Tested Separately and in Double Arrays

Layer[a]	Approximate Density ($g \cdot cm^{-3}$)	Thickness (mm)
White bread	0.25	19
Country white bread	0.22	13
Dense white bread	0.28	20
Pumpernickel bread	0.27	27
Rye bread	0.18	12
Pound cake	0.36	24

[a]The diameter of all of the samples was 30 mm.

MATERIALS AND METHODS

Pound cake, three kinds of white bread (country and dense), and pumpernickel and rye breads were purchased at a local supermarket. Cylindrical specimens were bored out of slices of these products with a cork borer, and their dimensions were determined with a caliper. The approximate density and dimensions of the specimens are listed in Table I. The specimens were tested either individually or in double-layered arrays. They were compressed between lubricated parallel plates at a speed of 10 mm $\cdot$ min^{-1}, using an Instron Universal Testing Machine model 1000. The Instron was interfaced with a Macintosh II computer (4MB RAM and 80MB disk) through an ACM2-12 interface board (Strawberry Tree Inc., Sunnyvale, CA). Computer software developed by M. D. Normand was used for control of the machine during the data acquisition process and for subsequent data processing. The latter included conversion of the machine's voltage versus real time output to stress-strain or strain-stress relationships with any desired stress and strain definitions, and it also included a conversion of a variety of mathematical models to the data, using nonlinear regression. In this work, two models in their $\epsilon = \epsilon(\sigma)$ form were selected for the regression, using the engineering strain or Hencky's strain (Swyngedau et al 1991a). These were model A,

$$\epsilon(\sigma) = \frac{C_{3i}\left(\frac{\sigma}{C_{1i}}\right)^{1/C_{2i}}}{1 + \left(\frac{\sigma}{C_{1i}}\right)^{1/C_{2i}}} \quad \text{or} \quad \sigma(\epsilon) = C_{1i}\left[\frac{\epsilon}{C_{3i} - \epsilon}\right]^{C_{2i}} \tag{7}$$

and model B,

$$\epsilon(\sigma) = C_{3i}[1 - \exp - (C_{1i}\sigma)]^{1/C_{2i}}$$

$$\text{or} \tag{8}$$

$$\sigma(\epsilon) = -\frac{1}{C_{1i}}\ln\left[1 - \left(\frac{\epsilon}{C_{3i}}\right)^{C_{2i}}\right]$$

As the mathematical structure of these two models indicate, C_1 is basically a scale factor with stress units in model A and compliance (stress^{-1}) units in model B. The prominence of the "shoulder" in the sigmoid stress-strain curve is expressed by the magnitude of C_2: when $C_2 = 1$ there is no "shoulder" at all. This relationship is monotonous because, as $\epsilon \to C_3$, in both models $\sigma \to \infty$ and C_3 is a measure of densification in terms of an asymptotic (dimensionless) strain. Alternatively, the smaller the value of C_3, the steeper is the stress-strain relationship at the densification region. The nonlinear regression itself was performed with the SYSTAT package (Wilkinson 1987), which also was used for plotting the results. All of the mechanical tests were performed in duplicates or triplicates, with a fresh sample in each test.

RESULTS AND DISCUSSION

Models A and B (equations 7 and 8), despite their mathematical dissimilarity, were equally appropriate, and their fit to the compression data, as judged by the magnitude of the χ^2 (Table II), was hardly affected by either the engineering strain or Hencky's strain (Figs. 2–5). This confirms previous reports of the behavior of both bread and synthetic solid foams (Swyngedau et al 1991a,b).

The constants C_1, C_2, and C_3 calculated from nonlinear regression of the compression data are listed in Table II. The constant C_3 was about the same in all of the tested materials, about 0.7 in terms of the engineering strain and a corresponding 1.2–1.3 in Hencky's strain. The magnitudes of C_1 and C_2 varied considerably, indicating differences in overall compressibility and in the extent to which the stress-strain relationships were sigmoid.

The regression data also were used to predict the compressive behavior of various double-layered arrays, using the procedure described in equations 1-6 for a case of two layers (1 and 2) only. We substituted the equation of model A or B with the corresponding coefficients (C_1, C_2, and C_3) for the terms $\epsilon_{E1}(\sigma)$ and $\epsilon_{E2}(\sigma)$ in equation 5 or $\epsilon_{H1}(\sigma)$ and $\epsilon_{H2}(\sigma)$ in equation 6, depending on whether the strain was engineering or Hencky's, and inserted the actual values of the initial height of H_{01} and H_{02} (see Fig. 1). (The subscripts 1 and 2 refer to the top and bottom layers, respectively.) Comparison of the predicted with the experimentally observed strain-stress behavior of double-layered arrays is shown in Figures 6-8. As could be expected, the strain-stress data of the double-layered array could be fitted by the very same models A and B that were used to describe each component, again irrespective of whether the strain is engineering or Hencky's.

As can be seen from Figures 6-8, and from the magnitude of the χ^2 values (Table II), the goodness of fit of models A and B to the double-layer compression data was about equal to that of the individual layers and occasionally even better. This indicates that the layered combinations of these spongy bakery products had the same characteristic compressive behavior as that of a single product and that the differences in the compressibility patterns are quantitative but not qualitative.

It also can be seen, Figures 6-8, that the predicted stress-strain relationships using equations 5 or 6, i.e., with either strain definition, were reasonably close to those actually observed. Therefore, the main assumptions on which the procedure is based—that the magnitude of lateral forces, if any, is negligible and therefore the stress in the two layers is practically the same, and that rate effects are not a major factor—were justified at least for the tested products. The fact that two very different mathematical expressions and strain definitions yielded the same results indicates that the predictions are not a coincidence but a manifestation of the characteristic compression mechanism of spongy materials. Because over a wide range of the strain the compression is dominated by cell wall collapse (Ashby 1983), there is little lateral expansion and consequently hardly any lateral stresses.

The slight discrepancy between the predicted and observed compressive behavior was most probably due to textural

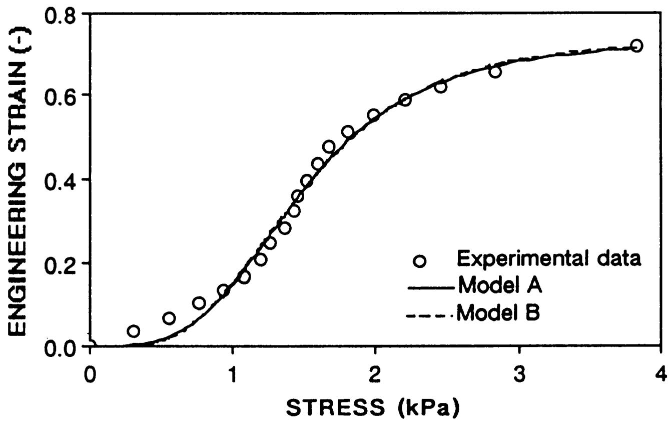

Fig. 2. Experimental and fitted strain-stress relationships of white bread crumbs. Model A, equation 7; model B, equation 8.

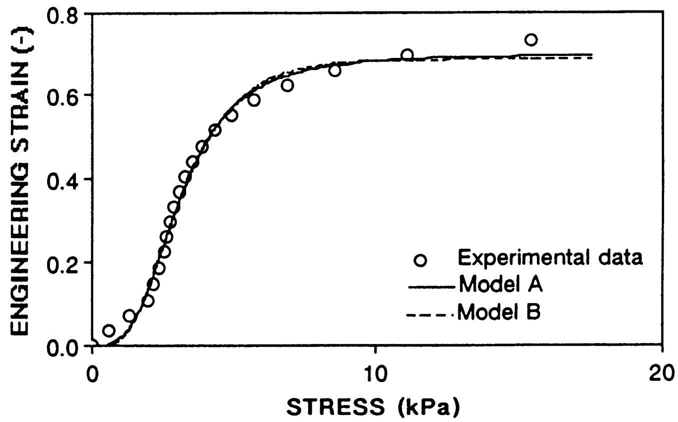

Fig. 3. Experimental and fitted strain-stress relationships of country white bread crumbs. Model A, equation 7; model B, equation 8.

TABLE II
Compressive Stress-Strain Characteristics of Various Spongy Baked Goods

		Model A (Eq 7)[b]				Model B (Eq 8)[b]			
Layer and Array	Strain Definition[a]	C_1 (kPa)	C_2 (−)	C_3 (−)	χ^2	C_1 (kPa^{-1})	C_2 (−)	C_3 (−)	χ^2
Dense white	E	2.7	0.41	0.70	0.007	0.60	0.33	0.68	0.010
Pumpernickel	E	3.8	0.37	0.69	0.007	0.48	0.26	0.67	0.009
Combined	E	3.5	0.41	0.72	0.003	0.46	0.33	0.70	0.005
Dense white	H	3.6	0.47	1.2	0.020	0.40	0.41	1.2	0.027
Pumpernickel	H	5.3	0.45	1.3	0.026	0.28	0.33	1.2	0.036
Combined	H	4.8	0.45	1.3	0.011	0.31	0.39	1.3	0.017
Country white	E	3.1	0.31	0.70	0.010	0.70	0.18	0.69	0.012
Rye	E	1.4	0.26	0.69	0.019	1.9	0.11	0.68	0.022
Combined	E	3.9	0.68	0.81	0.003	0.26	0.75	0.73	0.003
Country white	H	4.0	0.38	1.3	0.035	0.46	0.27	1.2	0.043
Rye	H	1.6	0.28	1.2	0.041	1.5	0.13	1.2	0.044
Combined	H	6.4	0.68	1.6	0.007	0.18	0.69	1.4	0.006
White	E	3.9	0.29	0.74	0.010	1.6	0.15	0.73	0.011
Pound cake	E	40,000	0.11	0.69	0.018	1.8	0.003	0.70	0.022
Combined	E	2,000	0.13	0.72	0.018	1.9	0.008	0.73	0.023
White	H	7.3	0.29	1.3	0.021	1.3	0.14	1.3	0.024
Pound cake	H	15,000	0.13	1.2	0.045	1.4	0.008	1.2	0.043
Combined	H	3,000	0.14	1.3	0.046	1.6	0.01	1.3	0.048

[a] E and H refer to the engineering strain and Hencky's strain definition, respectively.
[b] χ^2 is defined as $\Sigma\ [(O - E)^2/E]$ where O is the observed value, E is the expected value calculated from the model, and n the number of points. It was calculated from 210-230 data points. ($P \leq 0.01$ in all cases.) Since the magnitude of χ^2 in each row was calculated from the same set of data points, it can serve as a measure of goodness of fit of the two models.

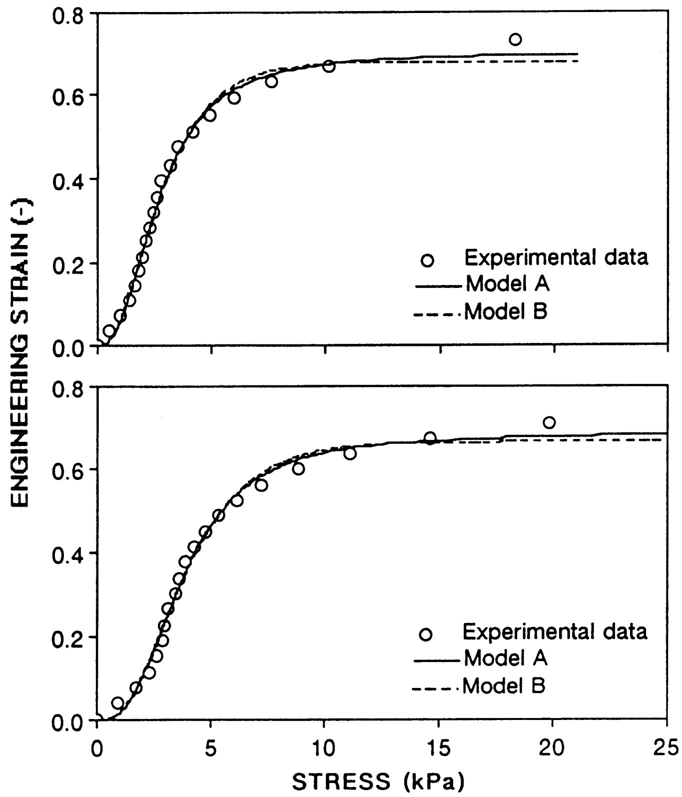

Fig. 4. Experimental and fitted strain-stress relationships of dense white bread crumbs (**top**) and pumpernickel bread crumbs (**bottom**). Model A, equation 7; model B, equation 8.

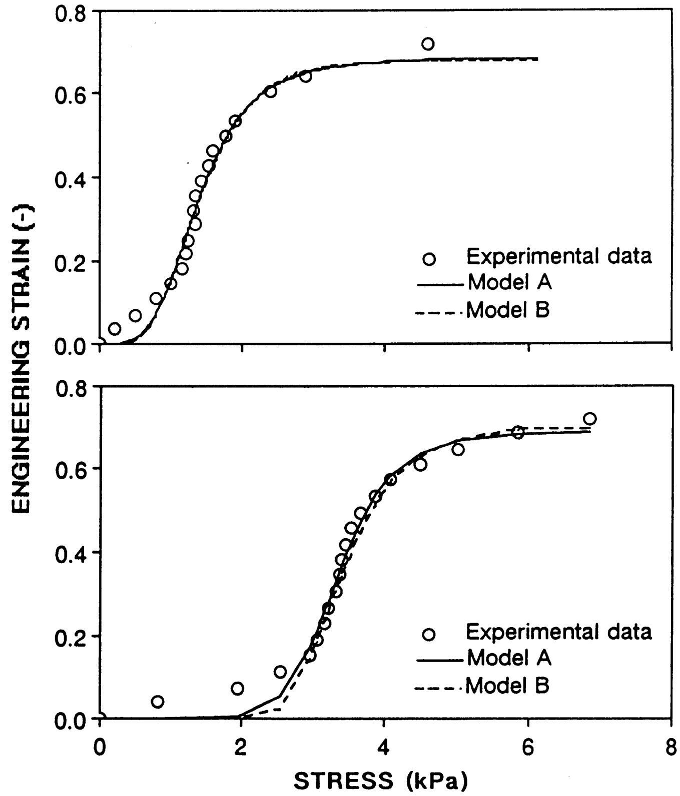

Fig. 5. Experimental and fitted strain-stress relationships of rye bread crumbs (**top**) and pound cake crumbs (**bottom**). Model A, equation 7; model B, equation 8.

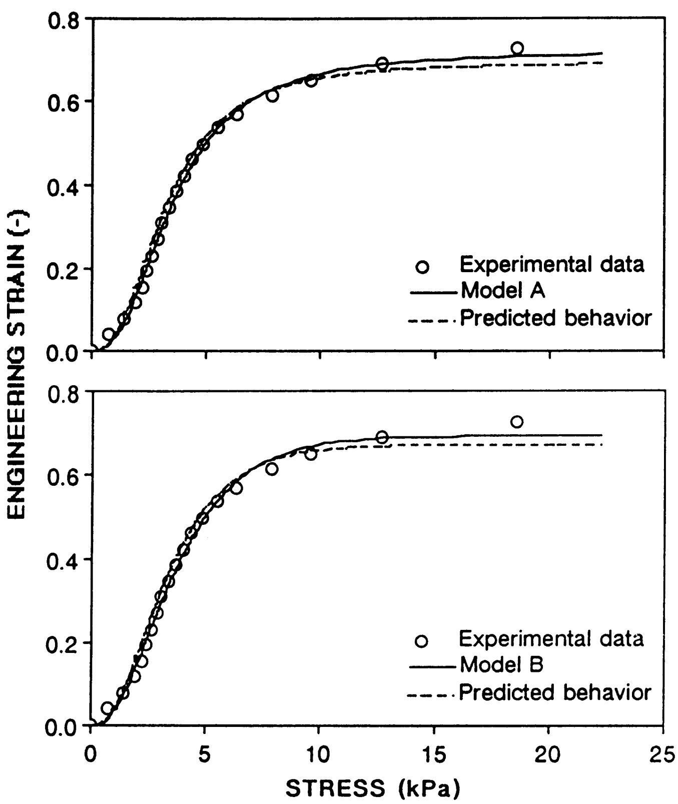

Fig. 6. Experimental, fitted, and predicted (equations 1–5) strain-stress relationships of a double-layered array of dense white and pumpernickel bread crumbs based on model A (**top**) and model B (**bottom**). For the regression results, see Table II.

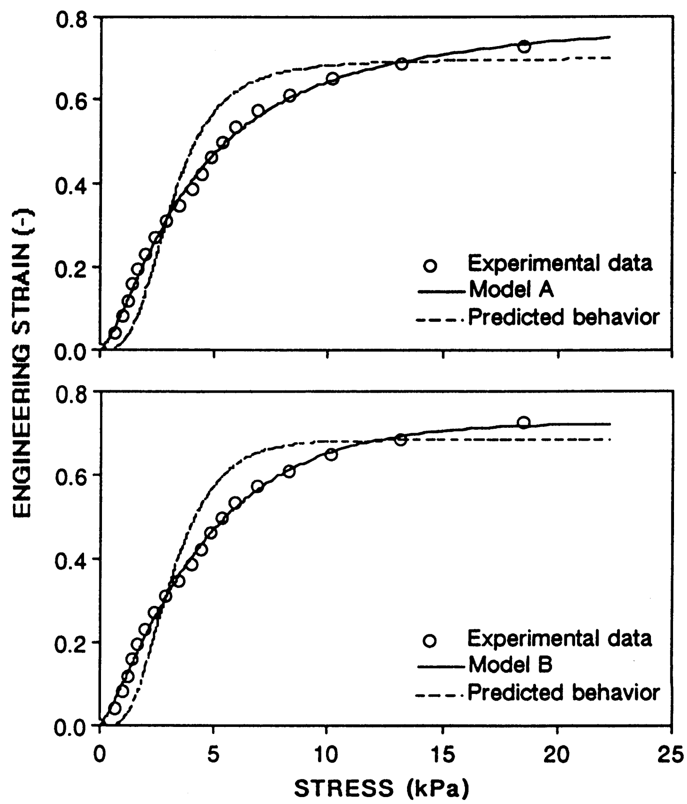

Fig. 7. Experimental, fitted, and predicted (equations 1–5) strain-stress relationships of a double-layered array of country white and rye bread crumbs based on model A (**top**) and model B (**bottom**). For the regression results, see Table II.

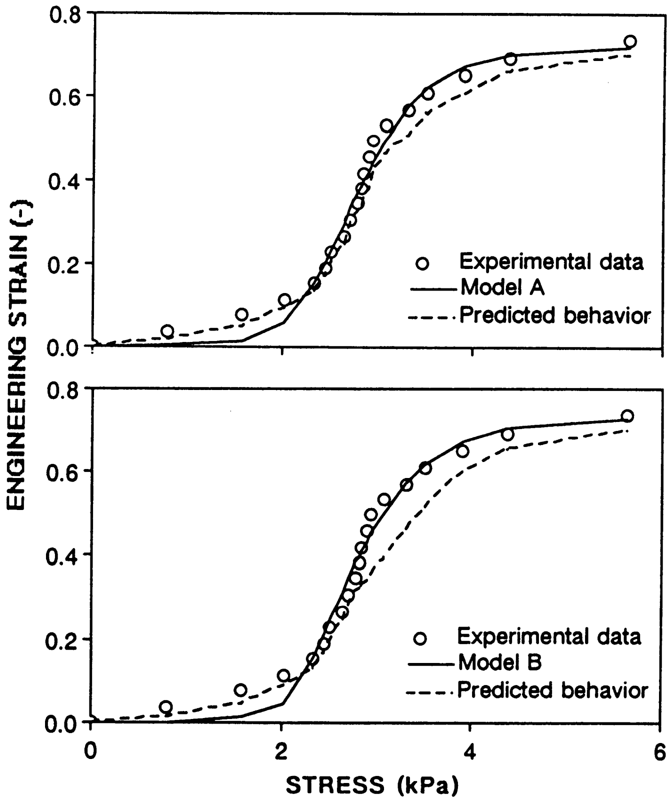

Fig. 8. Experimental, fitted, and predicted (equations 1-5) strain-stress relationships of a double-layered array of white bread and pound cake crumbs based on model A **(top)** and model B **(bottom)**. For the regression results, see Table II.

variability in the breads and cake and, to a much lesser extent, to the different strain rate histories of the individual layers and the arrays. The rate sensitivity of bakery products as well as that of many other food materials at strain rates on the order of 1-2 min^{-1} is very low and is frequently overshadowed by textural nonuniformity and imperfect specimen shape. If rate were a dominant factor, the observed strain of the array at every stress should have been consistently higher than the predicted. This was not the case in any of the tested arrays (Figs. 6-8). Our conclusions are supported by the fact that employing the same two models enabled a more accurate prediction of the behavior of double arrays of synthetic sponges, which are much more structurally uniform and physically stable (Swyngedau et al 1991a).

It should be added that the emphasis of this work was on sponge behavior in the deformation range, where the sigmoid shape of the stress-strain relationship is of paramount significance. In small deformations, the discrepancy between the predictions of the models and the observed values may be considerable, especially if expressed in relative terms. This, however, should not be considered a serious fault of the procedure or the models on which it is based. Under small deformation the stress-strain relationship can, for all practical purposes, be expressed by a linear relationship, and the problem of calculating an array behavior becomes trivial (Swyngedau et al 1991a). But even in such cases, the calculation is based on the assumptions that the force in the layers is the same, the deformations are additive, and small differences in the rate have negligible effect. The validity of these assumptions was demonstrated in this work for large deformations and is expected to remain in effect for smaller deformations, where the magnitude of lateral stresses would be even smaller and the strain rate is practically constant.

We only tested the applicability of the procedure in double arrays. In principle, the procedure should be applicable to multilayered arrays as long as the ratio of height to diameter is such that it will not induce bulking. It will be prudent, however, to verify the procedure with experimental results before it is applied to predict the behavior of multilayered systems such as multilayered cakes or similar products.

ACKNOWLEDGMENT

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LITERATURE CITED

ASHBY, M. L. 1983. The mechanical properties of cellular solids. Metall. Trans. 14A:1755-1769.

ATTENBURROW, G. E., GOODBAND, A. M., TAYLOR, L. J., and LILLFORD, P. J. 1989. Structure mechanics and texture of food sponge. J. Cereal Sci. 9:61-70.

GIBSON, L. J., and ASHBY, M. F. 1988. Cellular solids. Pergamon Press: New York.

MARIN, J. 1965. Mechanical relationships in testing for mechanical properties of polymers. Pages 87-146 in: Testing of Polymers, Vol. 1. J. V. Schmitz, ed. Interscience Publishing: New York.

PELEG, M., ROY, I., CAMPANELLA, O. H., and NORMAND, M. D. 1989. Mathematical characterization of the compressive stress-strain relationship of spongy baked goods. J. Food Sci. 54:947-949.

ROY, I., CAMPANELLA, O. H., NORMAND, M. D., and PELEG, M. 1989. Uniaxial compression of double layers of solid foods. J. Texture Stud. 20:443.455.

SWYNGEDAU, S., NUSSINOVITCH, A., and PELEG, M. 1991a. Models for the compressibility of layered polymeric sponges. Polym. Eng. Sci. 31:140-144.

SWYNGEDAU, S., NUSSINOVITCH, A., ROY, I., PELEG, M., and HUANG, V. 1991b. Comparison of four models for the compressibility of breads and plastic foams. J. Food Sci. 56:756-759.

WILKINSON, L. 1987. Systat: The System for Statistics. SYSTAT, Inc.: Evanston, IL.

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