Application of Muller's Method to Extensigraph Measurements with Various Hook Speeds[1]

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ABSTRACT

Muller's method for calculating the stress and strain for doughs from measurements with the Brabender Extensigraph was modified for use with different hook speeds. In the modified formula for stress, the effective mass and cross-sectional area of the dough test piece are expressed as a function of the length of the dough piece. With this modification, it was possible to use one equation for all mass and cross-sectional area calculations for all hook velocities. The resulting stress-strain curves were transformed to deformation and flow curves in the usual manner. Results were obtained for doughs from Canadian hard red spring wheat flour for the range of stresses from 7,500 to 40,000 $g.cm.^{-1} sec.^{-2}$. Flow curves showed that over this range of stresses the viscosity remained essentially constant for doughs mixed for 2 min., and increased slightly with increasing stress for doughs mixed for 9 and 15 min. Doughs rested for different periods differed in rheological behavior in the region of higher stress; doughs that were rested longer were less viscous.

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For fundamental measurement of rheological properties of dough, two experimental approaches have been used: measurement of shear deformation (1) and measurement of resistance in simple extension (2). Although extension is the simplest type of deformation, the use of extension data obtained with the Brabender Extensigraph in fundamental rheological equations is not simple. One approach for handling extensigraph data that appears promising is that of Muller et al. (2). The present paper deals with a modification of the equations for viscous properties of dough developed by Muller, and the application of the modified equation to data obtained with the Brabender Extensigraph using various hook speeds.

MATERIALS AND METHODS

The dough used to obtain data for the formulation and verification of the rheological equations was prepared in the 300-g. farinograph mixer using 300 g. flour and 194.4 ml. water. The flour was an untreated, straight-grade commercial flour of 13.1% protein and 0.42% ash, milled from Canadian hard red spring wheat. Three mixing times (in air)—2, 9, and 15 min.—were used to prepare doughs for the present study. The doughs were shaped 10 min. after mixing and stretched after an additional rest period of 5 min., except where noted otherwise. The hook was covered with a piece of Teflon sheeting to prevent the dough from sticking to it. Each test piece was stretched to an extension of about 10 cm. The data used in the calculations were read off the recorded curve.

The extensigraph was modified so that the speed of the hook could be varied from 0 to 110% of the normal speed. Hook speeds used were from 0.06 to 1.71 cm.sec.^{-1}. Each extensigraph curve was replicated three times and the average curve was used for the final calculations.

RESULTS AND DISCUSSION

Equation for Strain

At the outset, strain, e, was expressed as by Muller et al. (2):

$$e = \frac{l - l_o}{l_o} \tag{1}$$

where l_o is the half-length, in cm., of unstretched dough (1.875 cm.), and l is the half-length, in cm., of stretched dough at time, t. The half-length, l, can be calculated from the distance, h, travelled by the hook.

$$l = [h^2 + (3.75/2)^2]^{1/2} \tag{2a}$$

or

$$l = (h^2 + 3.52)^{1/2} \tag{2}$$

where 3.75 is the distance, in cm., between the lower ridges of the dough cradle (2 X 1.875). The distance travelled by the hook can be expressed as

$$h = 1.55V_hE - 0.0021R \tag{3}$$

where V_h is the velocity of the hook in $cm.sec.^{-1}$; E is length of the chart in cm.; R is resistance of the dough in g.; 1.55 is the time for one unit length of chart in $sec.cm.^{-1}$; and 0.0021 is a constant for correction of resistance for cradle depression in $cm.g.^{-1}$.

By substitution, the equation for strain, e, becomes

$$e = \frac{[(1.55V_hE - 0.0021R)^2 + 3.52]^{1/2} - l_o}{l_o} \tag{4}$$

Equation for Stress

The definition of stress of Muller et al. (2), as the tension per unit area, was adopted. Accordingly, the extension stress, σ, in $g.cm.^{-1} sec.^{-2}$, can be written as

$$\sigma = \frac{1}{3} \cdot \frac{T \cdot g}{A} \tag{5}$$

where T is the tension in g.; g is acceleration of gravity in $cm.sec.^{-2}$; and A is the area of the cross-section of the dough test piece.

This equation was adopted for the present study. If the value of T derived by Muller et al. (2) as

$$T = \frac{(2F + M)l}{4h}$$

is substituted in equation 5, the equation for stress becomes

$$\sigma = \frac{(2F + M)\, l \cdot g}{12A \cdot h} \tag{6}$$

where F represents force in g. (calibrated to equal 1.017R where R is the resistance in B.U.,) and M is the mass of dough. The difference between the treatment of Muller et al.(2) and that used in the present study is in the expression of A as a function of measurable parameters (see below).

Calculation of the Dough Mass, M

The effective mass of the dough increases significantly during extension. Using only one hook velocity, Muller et al. (2) expressed effective mass as a function of resistance to extension. This expression cannot be generalized for different hook speeds, and therefore it became necessary to express the mass as a function of the length of the dough piece. For each extension, the effective dough mass was determined by weighing. The plot of mass, M, against extensigraph resistance, R, (Fig. 1) gives what appears to be a sigmoid curve. Different curves are obtained for different hook speeds. A linear relationship between mass and resistance to extension, as adopted by Muller et al. (2), appears to be a reasonably good approximation for a specific hook speed. However, for a more general treatment of data obtained with different hook speeds, it was necessary to express the mass as a function of length of the test piece. Accordingly, the data of Fig. 1 are replotted in

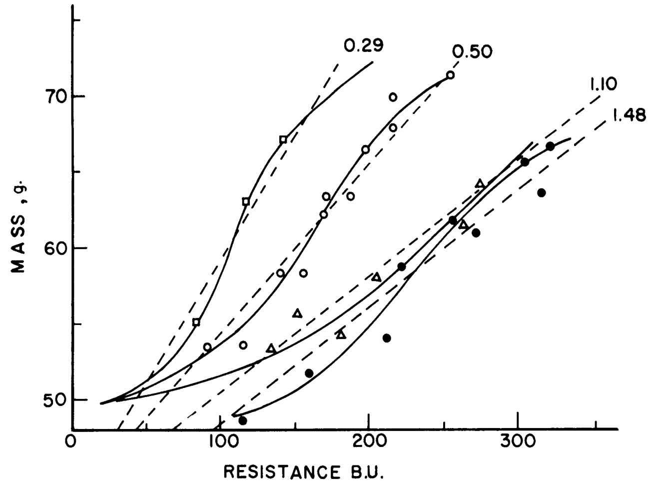

Fig. 1. Dependence of dough mass on resistance to extension for different hook speeds. Speed indicated in cm.sec.$^{-1}$.

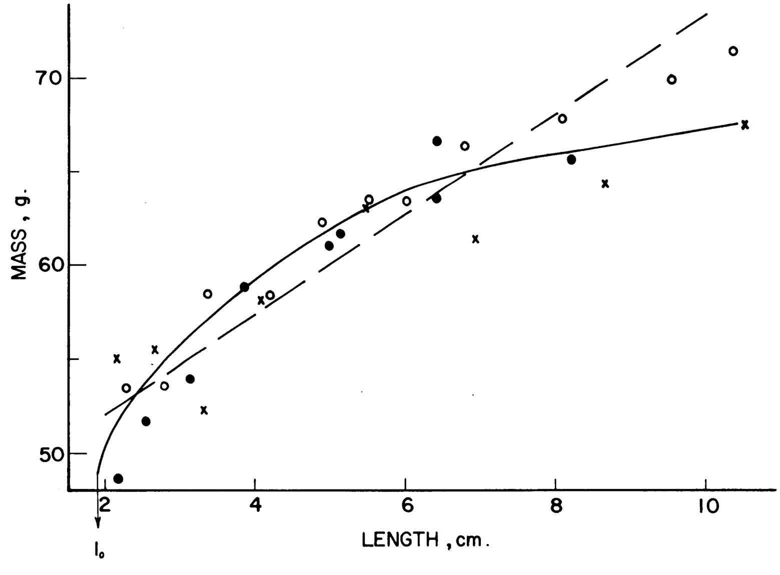

Fig. 2. Dependence of mass of dough strand on its length. The curve is the average of parabolas obtained for the four hook speeds and the line is the statistical regression line through all the points.

this manner in Fig. 2. Although the points show considerable scatter, the relationship can be expressed by a single parabolic function, or with somewhat less accuracy by a straight-line equation. The statistical straight line for the data shown in Fig. 2 is:

$$M = 47.26 + 2.39 \cdot l \qquad (7)$$

The use of parabolic and linear functions for expressing the relationship of dough mass to length in the construction of the stress-strain curves is examined later (see Fig. 6).

Determination of the Area of the Cross-Section, A

The determination of the cross-sectional area, required for the calculation of stress, was checked. Muller et al. (2) assumed that the area was inversely proportional to the length of the dough strand and used the formula,

$$A = \frac{M}{2\, l \cdot d} \qquad (8)$$

where d is the density of the dough. However, examination of the dough piece during the test indicates that the cross-sectional area is definitely non uniform along the strand (see Fig. 3 for diagrammatic representation). The deviations become particularly large for strains greater than one. The validity of equation 8 was checked by comparing calculated values with those obtained by direct measurement (Fig. 4). Although the four sets of measurements show considerable scatter, there is a definite deviation of the measured values from those calculated by equation 8.

The measured values of cross-sectional area for different strand lengths shown in

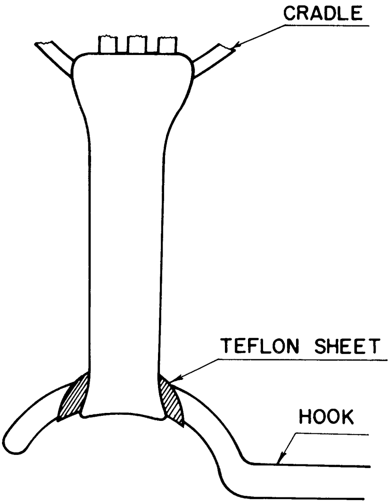

Fig. 3. Schematic of dough-strand shape during stretching.

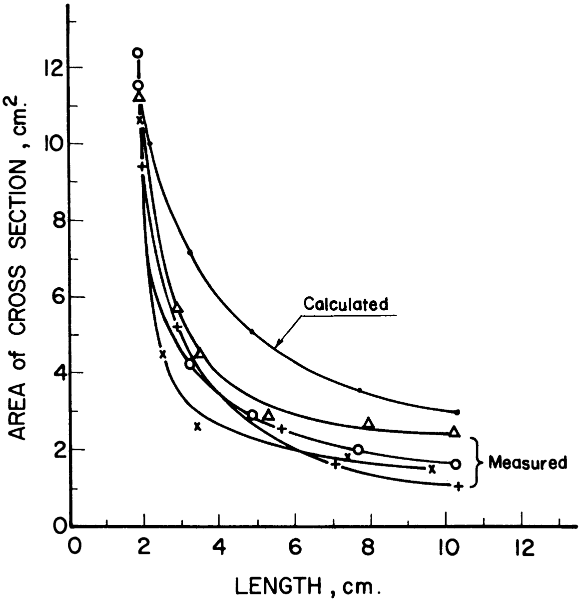

Fig. 4. Dependence of cross-sectional area of the dough strand on its length. Calculated curve was obtained by the method of Muller et al. (2) and the measured curves represent data obtained by direct measurement for four hook speeds in the range 0.29 to 1.48 cm.sec.$^{-1}$.

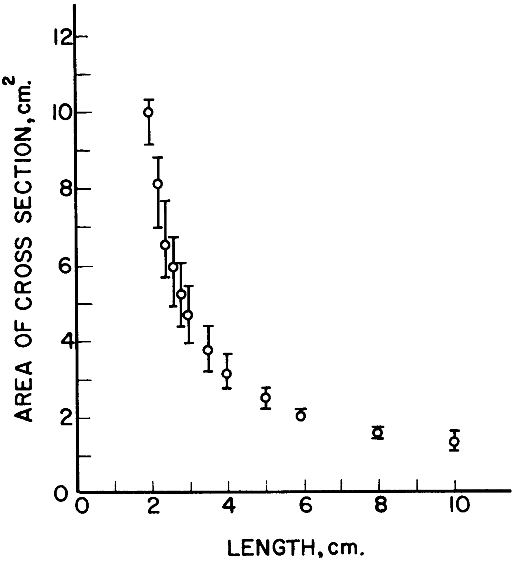

Fig. 5. Dependence of cross-sectional area of the dough strand on its length. The vertical lines show the variation of values obtained by a separate equation for each hook speed; the points were calculated from the equation with constants calculated as the average of the constants in the four equations for hook speeds from 0.29 to 1.71 cm.sec^{-1}.

Fig. 4 were used to calculate the best-fit curve for the data. The hyperbolic function obtained is:

$$A = 0.44 + \frac{7.79}{l - 1.19} \tag{9}$$

Figure 5 compares various calculated values of the cross-sectional area, A. The vertical lines represent the range of values obtained by use of a separate equation for each hook speed. The points were calculated from the statistically derived general equation 9, which gives a reasonably good representation of the data for the range of hook speeds investigated.

Modified Equation for Stress

To derive the equation for stress as a function of extensigraph resistance, R, and length, l, the following expressions were substituted into equation 6:

$F = 1.017R$ (obtained by calibration)
$M = 47.26 + 2.39 \cdot l$ (from equation 7)
$A = 0.44 + \frac{7.79}{l - 1.19}$ (from equation 9)

$h = (l^2 - 3.52)^{1/2}$ (from equation 2)

The new equation for stress, σ, becomes:

$$\sigma = \frac{(166R + 3{,}860 + 195 \cdot l)l}{\left(0.44 + \frac{7.79}{l - 1.19}\right)(l^2 - 3.52)^{1/2}} \tag{10}$$

Equation 10 can be readily modified to give stress, σ, as a function of extensigraph resistance and extensibility or resistance and strain, but the resulting equations are much more complex. The applicability of equation 10 to doughs other than those used in the present investigation remains to be verified by experimentation. Presumably the constants related to mass and cross-sectional area derived from equations 7 and 9 would depend on the type of dough used.

Figure 6 gives an example of a stress-strain curve obtained from equation 10 for a simple flour-water dough. The particular curve shown was obtained when the hook speed was 1.48 cm.sec.^{-1}, the normal speed of the extensigraph. The range of values of stress indicated by the vertical lines was obtained using four different equations (indicated in figure caption) for calculating mass as a function of the length of the test piece. The data for this hook speed were selected as the example because they showed the widest spread in stress values. On the basis of this graphical analysis, it was concluded that equation 10 is generally applicable for all hook speeds used.

The average (three experiments) stress-strain curves for six hook speeds in the range 0.06 to 0.90 cm.sec.^{-1} are shown in Fig. 7. For higher hook speeds up to 1.71 cm.sec.^{-1}, the stress-strain curves fall in the region between the curves of 0.71 and 0.90 cm.sec.^{-1} and are not shown. These curves are somewhat different from

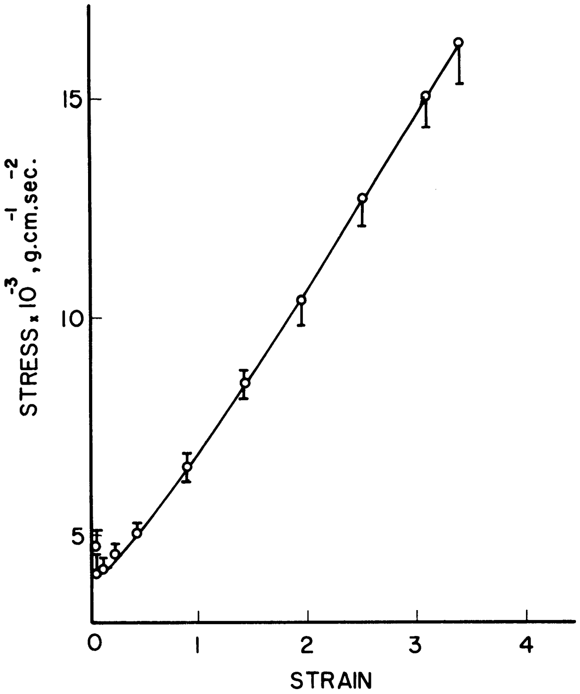

Fig. 6. Stress-strain curves for hook speed of 1.48 cm.sec.$^{-1}$. Each vertical line includes points calculated in four ways: 1) from the equation for mass of Muller et al.(2); 2) from the derived equation of parabola for this speed; 3) from the approximate straight-line equation for the experimental points for this speed; and 4) from the average straight line (Fig. 2) for all speeds. The points shown represent measured values.

those of Muller et al. (2), which showed a definite minimum in the region of low strains.

Formulation of Deformation and Flow Curves

Deformation curves show the relationship between the length of the dough strand and the time of extension under constant stress. The stress-strain curves were used to calculate the deformation curves. Data for these curves can be determined as follows: The values for length, l, are calculated from equation 1 using the value of strain, e, obtained from the stress-strain curve for a specific value of stress. The time, t, was calculated from equation 11 (below).

$$t = \frac{[(1.875e + 1.875)^2 - 3.52]^{\frac{1}{2}}}{V_h} \qquad (11)$$

Deformation curves for six values of stress in the range 7.5 X 10^3 to 40 X 10^3 g.cm.$^{-1}$ sec.$^{-2}$ for doughs mixed for 2 min. are shown in Fig. 8. Similar families of curves were obtained for the doughs mixed for 9 and 15 min. The shape of the deformation curves is typical for viscoelastic materials. The linear part of this curve represents viscous flow. The coefficient of viscous traction, λ, which is equal to

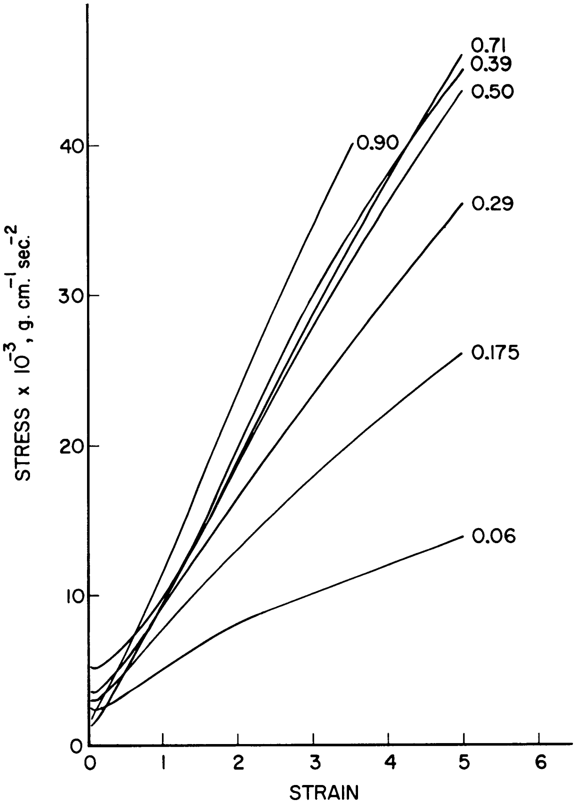

Fig. 7. Stress-strain curves for different hook speeds. Numbers indicate speed in cm.sec.$^{-1}$.

three times the coefficient of absolute viscosity in poise (3), can be calculated as the quotient of stress and relative velocity of extension from the slope of the linear part:

$$\lambda = \frac{\sigma}{\text{slope}/l_0} \tag{12}$$

The range of values for the coefficient of viscous traction obtained for various doughs examined in the present investigation is from 5.5 X 10^5 to 11.2 X 10^5 g.cm.$^{-1}$ sec.$^{-1}$. These values exceed those of Muller et al. (3) by an order of magnitude. The difference is attributed to the difference in intrinsic properties of the flours used in the two studies.

Flow curves show the relationship of the relative velocity of viscous deformation (slope of linear portion of deformation curve) to stress. Flow curves for flour-water doughs mixed for 2, 9, and 15 min. are shown in Fig. 9. The data for the 2-min. dough were derived from the deformation curves in Fig. 8; deformation curves for the other two flow curves are not shown. These curves show that the rheological behavior of the 2-min. dough is essentially linear in the range of stresses

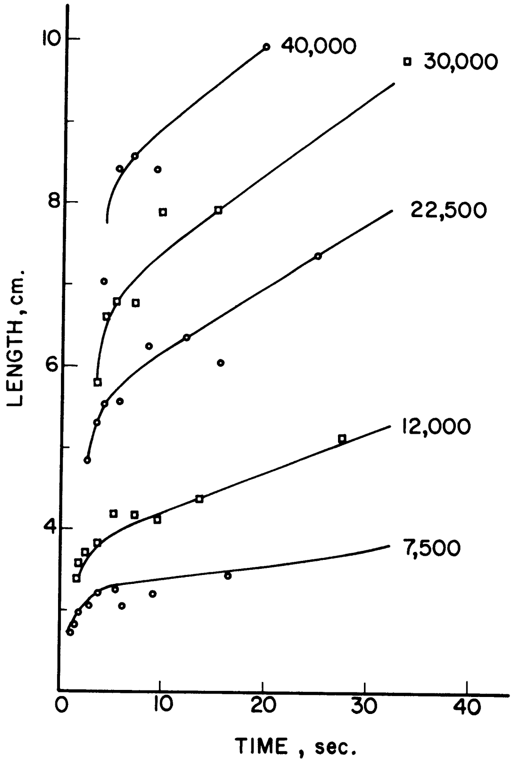

Fig. 8. Deformation curves for the dough mixed for 2 min. Numbers indicate the stress in g.cm.$^{-1}$sec.$^{-2}$.

investigated. The other two doughs showed a slight non linear behavior; the viscosity appeared to decrease with increasing stress.

Flow curves analogous to those of Fig. 9 can be constructed to examine the rheological behavior of any type of dough. The effect of resting the dough at a constant temperature of 30°C. from 15 to 120 min. is shown in Fig. 10 by way of example. Over the range of stresses examined, the flow behavior is non linear. For stresses above 15 X 10^3 g.cm.$^{-1}$sec.$^{-2}$, the doughs which were rested for longer times were characterized by higher slopes of the flow curves and lower coefficients of viscous traction.

GENERAL DISCUSSION

The present investigation represents an extension of an earlier study of Muller et al. (2) on the use of the Brabender Extensigraph as a fundamental rheological instrument. By modifying the Extensigraph to provide different hook speeds, it was possible to obtain data which could be used to construct the flow curve for a particular dough. This curve depicts the rheological behavior of the dough for a specific range of stresses. This approach should prove useful for characterizing

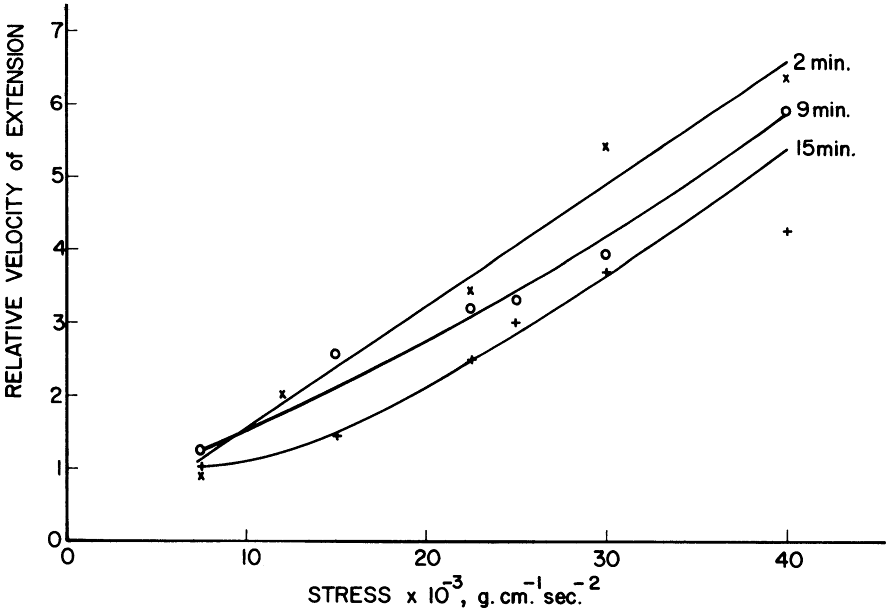

Fig. 9. Flow curves for doughs mixed for 2, 9, and 15 min.

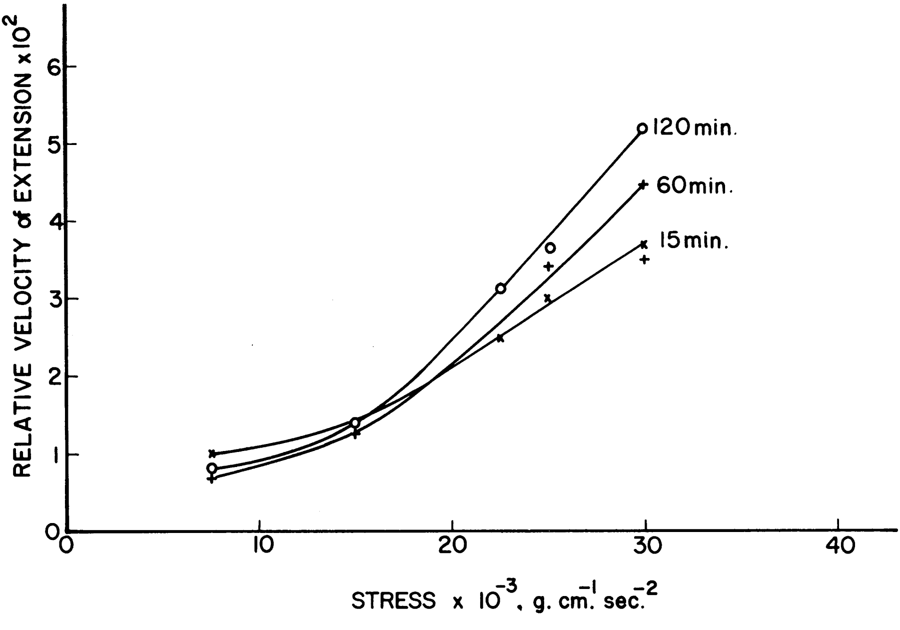

Fig. 10. Flow curves for the 15-min. dough stretched 15, 60, and 120 min. after mixing.

technologically important parameters in terms of fundamental rheological properties.

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